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Temperatures of circular rods due to rotational and oscillatory axial sliding motion

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Abstract — Using Green's function method, surface temperature rise due to frictional heating in oscillatory sliding and rotating rods is studied. The problem under consideration consists two rods, one of them is stationary and the other exhibits both oscillatory axial sliding and rotational motion. In terms of the obtained closed form solutions for the temperature distribution in both rods, the effect of different parameters on the rod thermal behavior may be investigated. © 2001 Éditions scientifiques et médicales Elsevier SAS

frictional heating / dry friction / conduction heat transfer / temperature distribution / rotating and oscillating rods

Nomenclature

Bi	Biot number $= hr_0/k$	
c	specific heat	$J{\cdot}kg^{-1}{\cdot}K^{-1}$
$\overline{E}(t)$	axial location of the moving frictional heating source	m
$E(\tau)$	dimensionless axial location of the source $= \overline{E}(t)r_0^{-1}$	
F_1, F_2	fractions of frictional heating absorbed by the sliding and stationary rods, respectively	
g	frictional heating source term	$W{\cdot}m^{-3}$
h	convective heat transfer coefficient	$W{\cdot}m^{-2}{\cdot}K^{-1}$
h_c	convective heat transfer coefficient at the contact region	
h_0	length of the sliding rod	m
h_1, h_2	axial boundaries of the moving frictional heating source	
h_3	axial location of the frictional heating source on the stationary rod	m
h_4	length of the stationary rod	m
H_0	dimensionless length of the sliding rod $= h_0/r_0$	
H_1, H_2	dimensionless axial boundaries of the moving frictional heating source $= h_1/r_0$, h_2/r_0	
H_3	dimensionless axial location of the frictional heating source on the stationary rod $= h_3/r_0$	
H_4	dimensionless length of the stationary rod $= h_4/r_0$	
k	thermal conductivity	$W{\cdot}m^{-1}{\cdot}K^{-1}$
P	contact pressure	Pa
r	radial coordinate	m
r_0	radius of the sliding rod	m
r_1	radius of the stationary rod	m
R	dimensionless radial cooordinate $= r/r_0$	
R_1	dimensionless radius of the stacionary rod $= r_1/r_0$	
t	time	s
t_0	period of the sliding motion	s
T	temperature	K
T_i	initial temperature	K
T_∞	ambient temperature	K
V	absolute speed of the moving source	$m{\cdot}s^{-1}$
V_z	axial speed component of the moving source	$m{\cdot}s^{-1}$
z	axial coordinate	m
Z	dimensionless axial coordinate $= z/r_0$	

Greek symbols

α	thermal diffusivity	$m^2{\cdot}s^{-1}$
α_R	thermal diffusivity ratio $= \alpha_1/\alpha_2$	
δ	delta function	

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Δr, Δz, $\Delta\phi$	the dimensions of the contact region	m
θ	dimensionless temperature $= (T - T_\infty)/T_\infty$	
μ	friction coefficient	
τ	dimensionless time $= t\alpha_1/r_0^2$	
τ_0	dimensionless period of the sliding motion $= t_0\alpha_1/r_0^2$	
ϕ	angular coordinate	rad
ϕ_0	parameter defined as $= \omega r_0^2/\alpha_1$	
ω	angular velocity of the rotating rod .	rad·s^{-1}

Subscripts

1	sliding rod
2	stationary rod
$1l$, $2l$	left sides of the sliding and stationary rods, respectively
$1s$, $2s$	circumferential sides of the sliding and stationary rods, respectively
$1u$, $2u$	right sides of the sliding and stationary rods, respectively
∞	ambient conditions

1. INTRODUCTION

The effect of temperature, due to friction, on materials is quite substantial. In recent years, different investigations [1–4] have been carried out to explain the temperature processes which proceed during frictional contact. Most of these investigations have been carried out for unidirectional sliding systems. However, many sliding contacts occur in oscillatory sliding systems, such as reciprocating seals or polymer sleeve bearings for business machines or aircraft control systems. The wear behavior of low melting point materials, such as polymers, in oscillatory contact has been found to be different from that in unidirectional sliding [5]. As in the case of unidirectional sliding, the coefficient of friction and wear rate of polymer materials in oscillatory sliding are significantly influenced by the surface temperature [6]. There is, therefore, a need to be able to predict the sliding temperatures in such cases.

Using the theoretical approach initially outlined by Jaeger [7], Hirano and Yoshida [8] computed the surface temperature rise for a square heat source reciprocating on a semi-infinite medium. It is found that there existed a remarkable difference between the surface temperature rise in oscillatory sliding and that in unidirectional sliding, especially when the oscillating frequency and oscillation amplitude are high. The high temperature variation at the contact interface due to oscillatory sliding is suggested to be a major cause for severe fretting wear [9].

Tian and Kennedy [10] presented an analytical and experimental investigation of surface temperature rise at the contact interface for oscillatory sliding. The sliding objects are considered semi-infinite media. Other theoretical investigations are presented in [11, 12]. Experimental investigations to predict the contact temperature are carried out by [13, 14] using infrared scanning or infrared thermal imaging. The above-mentioned studies provide useful information about the characteristics of the contact temperature rise due to sliding. However, some of these studies present either numerical or experimental investigations. Others consider unidirectional sliding with no oscillation or consider reciprocating sliding motion in the axial direction, with no rotational motion. Also, some of the above-mentioned studies treat the sliding objects as a semi-infinite media.

In the present work, a simple mathematical model, with realistic assumptions, is presented to describe the thermal behavior of two circular rods sliding over each other. One of the two rods exhibits both an axial reciprocating sliding and a rotational motion. Also, the two rods may have small radii and, as a result, the semi-infinite domain assumption in the radial direction is not valid. The axes of the two rods are perpendicular. The mathematical model is presented in terms of two energy equations, one for each rod, coupled at the contact area. These equations are solved analytically using Green's function method. In terms of the obtained solution, the temperature distribution within the two rods and within the contact area may be found. Then the effect of the contact temperature on the friction coefficient may be featured.

2. MATHEMATICAL FORMULATION

As shown in *figure 1*, the system of interest consists of two perpendicular cylindrical rods where one rod is fixed (rod 2) and the other one (rod 1) is acting as a reciprocating rider in the axial direction and rotates in the same time. Now, referring to the same figure, the energy equations describing the thermal behavior of the two rods are given as

$$\frac{\partial^2 T_i}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 T_i}{\partial \phi^2} + \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r} + \frac{g_i}{k_i} = \frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} \quad \text{for } i = 1, 2 \qquad (1)$$

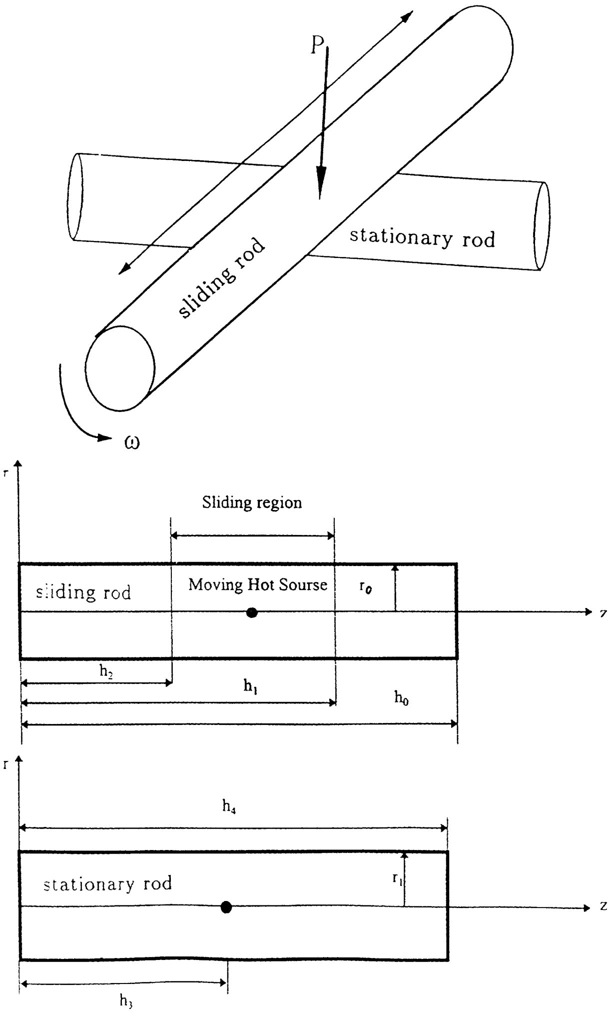

Figure 1. Schematic diagram of the problem under consideration.

where

$$g_1(t,r,z,\phi) = g_p^c \frac{F_1}{r}\bar{\delta}(r-r_0)\bar{\delta}(\phi-\omega t)\bar{\delta}\big(z-\overline{E}(t)\big)$$

$$g_2(t,r,z,\phi) = g_p^c \frac{F_2}{r}\bar{\delta}(r-r_1)\bar{\delta}(\phi)\bar{\delta}(z-h_3)$$

$$g_p^c = \mu P V r_0 \Delta\phi \Delta z, \qquad V = \sqrt{V_z^2 + \omega^2 r_0^2}$$

and F_1 and F_2 are the fractions of heat generated at the contact area due to friction and absorbed by the reciprocating and the stationary rods, respectively, V is the absolute speed of the moving source, V_z is the magnitude of the axial sliding speed, P is the contact pressure, ω is the angular velocity of the rotating rod, δ is the delta function, $\overline{E}(t)$ is a periodic function of any form, as shown in *figure 2*, which describes the

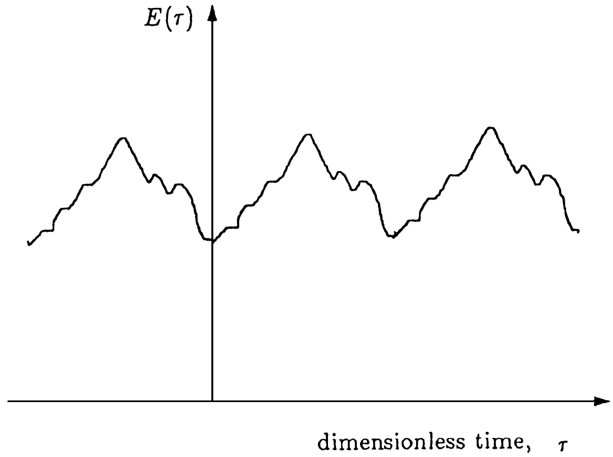

Figure 2. Periodic time variation in the location of the contact point.

location of the moving frictional heating source which moves between h_1 and h_2. This periodic function may be expanded, using Fourier series, as

$$\overline{E}(t) = \frac{\bar{a}_0}{2} + \sum_{n=1}^{\infty} \bar{a}_n \cos\frac{n\pi t}{t_0} \tag{2}$$

where $\bar{a}_0, \bar{a}_n$ are the coefficients of the Fourier expansion which values depend on the form of $\overline{E}(t)$, and they are given as

$$a_n = \frac{2}{t_0}\int_0^t \overline{E}(t)\cos\frac{n\pi t}{t_0}\,\mathrm{d}t$$

and t_0 is the period of the sliding motion. Equations (1) have the following initial and boundary conditions:

$$T_1(0,r,z,\phi) = T_2(0,r,z,\phi) = T_i \tag{3}$$

$$\frac{\partial T_1}{\partial r}(t,0,z,\phi) = \frac{\partial T_2}{\partial r}(t,0,z,\phi) = 0 \tag{4}$$

$$\frac{\partial T_1}{\partial r}(t,r_0,z,\phi) + \frac{h_{1s}}{k_1}T_1(t,r_0,z,\phi) = \frac{h_{1s}T_\infty}{k_1} \tag{5}$$

$$\frac{\partial T_2}{\partial r}(t,r_1,z,\phi) + \frac{h_{2s}}{k_2}T_2(t,r_1,z,\phi) = \frac{h_{2s}T_\infty}{k_2} \tag{6}$$

$$\frac{\partial T_1}{\partial z}(t,r,0,\phi) - \frac{h_{1l}}{k_1}T_1(t,r,0,\phi) = -\frac{h_{1l}T_\infty}{k_1} \tag{7}$$

$$\frac{\partial T_2}{\partial z}(t,r,0,\phi) - \frac{h_{2l}}{k_2}T_2(t,r,0,\phi) = -\frac{h_{2l}T_\infty}{k_2} \tag{8}$$

$$\frac{\partial T_1}{\partial z}(t,r,h_0,\phi) + \frac{h_{1u}}{k_1}T_1(t,r,h_0,\phi) = \frac{h_{1u}T_\infty}{k_1} \tag{9}$$

$$\frac{\partial T_2}{\partial z}(t,r,h_4,\phi) + \frac{h_{2u}}{k_2}T_2(t,r,h_4,\phi) = \frac{h_{2u}T_\infty}{k_2} \tag{10}$$

and F_1 and F_2 are determined from the following equations:

$$T_1\big(t, r_0, \overline{E}(t), \phi\big) = T_2(t, r_1, h_3, 0) \tag{11}$$

$$-k_1\frac{\partial T_1}{\partial r}\big(t, r_0, \overline{E}(t), \phi\big) - k_2\frac{\partial T_2}{\partial r}(t, r_1, h_3, 0) - h_c(T_c - T_\infty) + \mu PV = 0 \tag{12}$$

where $T_c = T_1(t, r_0, \overline{E}(t), \phi) = T_2(t, r_1, h_3, 0)$, h_c is the convective heat transfer coefficient at the contact area between the two rods. Equation (12) states that the generated energy due to friction is distributed among both rods and the ambient, and equation (11) states that both rods have the same temperature at their contact. It is worth mentioning here two points: (1) different workers rely on experimental values of F_1 and F_2. (2) We assumed in our analysis that the contact area is very small so it can be considered as a point. This assumption is valid in this analysis for the following reasons: the first reason is that the actual area of contact is small compared to the heat transfer area. For example, if we considered two steel rods of 16 in and 12 in diameters loaded with a force of 100 lb, according to Hertz's theory the area of contact is 0.0072 in^2 [15]. The second reason, the amount of heat generated due to friction can be calculated based on the actual area of contact and then introduced to the source term, S, as if the area of contact is a point.

Now, using the dimensionless parameters defined in the nomenclature, equations (1) are rewritten as

$$\frac{\partial^2\theta_1}{\partial Z^2} + \frac{1}{R^2}\frac{\partial^2\theta_1}{\partial\phi^2} + \frac{\partial^2\theta_1}{\partial R^2} + \frac{1}{R}\frac{\partial\theta_1}{\partial R} + \frac{S_1F_1}{R}\delta(R-1)\delta(\phi-\phi_0\tau)\delta\big(Z-E(\tau)\big) = \frac{\partial\theta_1}{\partial\tau} \tag{13}$$

$$\frac{\partial^2\theta_2}{\partial Z^2} + \frac{1}{R^2}\frac{\partial^2\theta_2}{\partial\phi^2} + \frac{\partial^2\theta_2}{\partial R^2} + \frac{1}{R}\frac{\partial\theta_2}{\partial R} + \frac{S_2F_2}{R}\delta(R-R_1)\delta(\phi)\delta(Z-H_3) = \frac{1}{\alpha_R}\frac{\partial\theta_2}{\partial\tau} \tag{14}$$

where

$$\theta_1\big(\tau, 1, E(\tau), \phi\big) = \theta_2(\tau, R_1, H_3, 0) \tag{15}$$

$$\frac{\partial\theta_1}{\partial R}\big(\tau, 1, E(\tau), \phi\big) + k_R\frac{\partial\theta_2}{\partial R}(\tau, R_1, H_3, 0) + Bi_c\theta_c - S_3 = 0 \tag{16}$$

with

$$\theta_c = \theta_1\big(\tau, 1, E(\tau), \phi\big) = \theta_2(\tau, R_1, H_3, 0)$$

$$\phi_0 = \frac{\omega r_0^2}{\alpha_1}$$

$$E(\tau) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi\tau}{\tau_0}$$

where

$$a_n = \frac{2}{\tau_0}\int_0^{\tau_0} E(\tau)\cos\frac{n\pi\tau}{\tau_0}\,\mathrm{d}\tau$$

$E(\tau)$ is the dimensionless periodic function which describes the dimensionless location of the contact area on the slider rod, and τ_0 is the dimensionless period of $E(\tau)$. Also,

$$S_1 = \frac{\mu PV\Delta\phi\Delta z}{k_1T_\infty}, \qquad S_2 = \frac{\mu PV\Delta\phi\Delta z}{k_2T_\infty}$$

$$S_3 = \frac{\mu PVr_0}{k_1T_\infty}, \qquad Bi_c = \frac{h_cr_0}{k_1}, \qquad k_R = \frac{k_2}{k_1}$$

The dimensionless initial and boundary conditions are given as

$$\theta_1(0, R, Z, \phi) = \theta_2(0, R, Z, \phi) = \theta_0$$
$$\frac{\partial\theta_1}{\partial R}(\tau, 0, Z, \phi) = \frac{\partial\theta_2}{\partial R}(\tau, 0, Z, \phi) = 0 \tag{17}$$

$$\frac{\partial\theta_1}{\partial R}(\tau, 1, Z, \phi) + Bi_{1s}\theta_1(\tau, 1, Z, \phi) = 0 \tag{18}$$

$$\frac{\partial\theta_2}{\partial R}(\tau, R_1, Z, \phi) + Bi_{2s}\theta_2(\tau, R_1, Z, \phi) = 0 \tag{19}$$

$$\frac{\partial\theta_1}{\partial Z}(\tau, R, 0, \phi) - Bi_{1l}\theta_1(\tau, R, 0, \phi) = 0 \tag{20}$$

$$\frac{\partial\theta_2}{\partial Z}(\tau, R, 0, \phi) - Bi_{2l}\theta_2(\tau, R, 0, \phi) = 0 \tag{21}$$

$$\frac{\partial\theta_1}{\partial Z}(\tau, R, H_0, \phi) + Bi_{1u}\theta_1(\tau, R, H_0, \phi) = 0 \tag{22}$$

$$\frac{\partial\theta_2}{\partial Z}(\tau, R, H_3, \phi) + Bi_{2u}\theta_2(\tau, R, H_3, \phi) = 0 \tag{23}$$

where

$$Bi_{1s} = \frac{h_{1s}r_0}{k_1}, \qquad Bi_{2s} = \frac{h_{2s}r_0}{k_2}$$

$$Bi_{1l} = \frac{h_{1l}r_0}{k_1}, \qquad Bi_{2l} = \frac{h_{2l}r_0}{k_2}$$

$$Bi_{1u} = \frac{h_{1u}r_0}{k_1}, \qquad Bi_{2u} = \frac{h_{2u}r_0}{k_2}$$

$$H_0 = \frac{h_0}{r_0}, \qquad H_3 = \frac{h_3}{r_0}$$

Now, following the analytical development given in Appendix, equations (13)–(23) are solved, using Green's function method, as

$$\theta_1(\tau,R,Z,\phi) = \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{e^{-(\beta_{1m}^2+\eta_{1p}^2\tau)}}{N(\beta_{1m})N(\eta_{1p})}J_0(\beta_{1m}R) \cdot \left[\eta_{1p}\cos\eta_{1p}Z + Bi_{1l}\sin\eta_{1p}Z\right]\theta_0 \cdot \left[\sin\eta_{1p}Z' - \frac{Bi_{1l}}{\eta_{1p}}\cos\eta_{1p}Z'\right]_0^{H_0}\frac{J_1(\beta_{1m})}{\beta_{1m}} + \int_{\tau^*=0}^{\tau}\frac{d\tau^*}{\pi}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=1}^{\infty}\frac{e^{-(\beta_{1m}^2+\eta_{1p}^2)(\tau-\tau^*)}}{N(\beta_{1m})N(\eta_{1p})} \cdot J_\nu(\beta_{1m}R)\left[\eta_{1p}\cos\eta_{1p}Z + Bi_{1l}\sin\eta_{1p}Z\right] \cdot J_\nu(\beta_{1m})\left[\eta_{1p}\cos\eta_{1p}E(\tau^*) + Bi_{1l}\sin\eta_{1p}E(\tau^*)\right] \cdot \cos(\nu\phi - \nu\phi_0\tau^*)S_1F_1 \tag{24}$$

$$\theta_2(\tau,R,Z,\phi) = \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{e^{-\alpha_R(\beta_{2m}^2+\eta_{2p}^2)\tau}}{N(\beta_{2m})N(\eta_{2p})}J_0(\beta_{2m}R) \cdot \left[\eta_{2p}\cos\eta_{2p}Z + Bi_{2l}\sin\eta_{2p}Z\right]\theta_0 \cdot \left[\sin\eta_{2p}Z' - \frac{Bi_{2l}}{\eta_{2p}}\cos\eta_{2p}Z'\right]_0^{H_4}\frac{R_1J_1(\beta_{2m}R_1)}{\beta_{2m}} + \frac{\alpha_R}{\pi}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=1}^{\infty}\frac{1-e^{-\alpha_R(\beta_{2m}^2+\eta_{2p}^2)\tau}}{N(\beta_{2m})N(\eta_{2p})\alpha_R(\beta_{2m}^2+\eta_{2p}^2)} \cdot J_\nu(\beta_{2m}R)\left[\eta_{2p}\cos\eta_{2p}Z + Bi_{2l}\sin\eta_{2p}Z\right] \cdot J_\nu(\beta_{2m}R_1)\left[\eta_{2p}\cos\eta_{2p}H_3 + Bi_{2l}\sin\eta_{2p}H_3\right] \cdot \cos(\nu\phi)S_2F_2 \tag{25}$$

Note that the first term in both equations (24) and (25) does not depend on ϕ. This is predicted, since the first term is resulted from the non-homogeneous term in the initial condition. However, in our case the initial condition does not depend on ν and, as a result, the first term will be axisymetric. If the experimental values of F_1 and F_2 are available, then they may be inserted directly into equations (24) and (25) to get θ_1 and θ_2. In another approach, F_1 and F_2 may be obtained by solving equations (15) and (16) in terms of equations (24) and (25).

3. SPECIAL CASE

Consider the case in which the moving rod exhibits a pure rotational motion, without axial reciprocating sliding motion. For this special case, $V_z = 0$, $E(\tau) = H_1$ and $V = \omega R_0$. As a result, equations (24) and (25) are reduced to

$$\theta_1(\tau,R,Z,\phi) = \frac{1}{\pi}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=0}^{\infty}\frac{J_\nu(\beta_{1m}R)}{N(\beta_{1m})N(\eta_{1p})} \cdot \left[\eta_{1p}\cos\eta_{1p}Z + Bi_{1l}\sin\eta_{1p}Z\right]J_\nu(\beta_{1m}) \cdot \left[\eta_{1p}\cos\eta_{1p}H_1 + Bi_{1l}\sin\eta_{1p}H_1\right]Q(t)S_1F_1 \tag{26}$$

$$\theta_2(\tau,R,Z,\phi) = \frac{\alpha_R}{\pi}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=0}^{\infty}\frac{1-e^{-\alpha_R(\beta_{2m}^2+\eta_{2p}^2)\tau}}{[N(\beta_{2m})N(\eta_{2p})]\alpha_R(\beta_{2m}^2+\eta_{2p}^2)} \cdot J_\nu(\beta_{2m}R)\left[\eta_{2p}\cos\eta_{2p}Z + Bi_{2l}\sin\eta_{2p}Z\right] \cdot J_\nu(\beta_{2m}R_1)\left[\eta_{2p}\cos\eta_{2p}H_3 + Bi_{2l}\sin\eta_{2p}H_3\right] \cdot \cos(\nu\phi)S_2F_2 \tag{27}$$

where π is replaced by 2π for $\nu = 0$, and

$$Q(t) = \Big\{\cos\nu\phi\left[\left(\beta_{1m}^2+\eta_{1p}^2\right)\cos\nu\phi_0\tau + \nu\phi_0\sin\nu\phi_0\tau\right] + \sin\nu\phi\left[\left(\beta_{1m}^2+\eta_{1p}^2\right)\sin\nu\phi_0\tau - \nu\phi_0\cos\nu\phi_0\tau\right] - \cos\nu\phi\left[\left(\beta_{1m}^2+\eta_{1p}^2\right)e^{-(\beta_{1m}^2+\eta_{1p}^2)\tau}\right] + \sin\nu\phi\left[\nu\phi_0 e^{-(\beta_{1m}^2+\eta_{1p}^2)\tau}\right]\Big\} \cdot \left\{\left[\beta_{1m}^2+\eta_{1p}^2\right]^2 + \nu^2\phi_0^2\right\}^{-1} \tag{28}$$

where π is replaced by 2π for $\nu = 0$.

4. RESULTS

It is not our intention in this section to carry out a parametric study for the problem under consideration. The results presented here are aimed at demonstrating the behavior of the obtained solution under different conditions. We will investigate the time variation in the dimensionless temperature of a fixed location at the outer surface of the rotating rod (rod 1) under different conditions. The values of $R = 1$, $Z = (H_1 + H_2)/2$, $Bi_{1l} = 0$, $Bi_{1s} = 0$, $\phi = 0$ are used in obtaining the solution for all cases presented here.

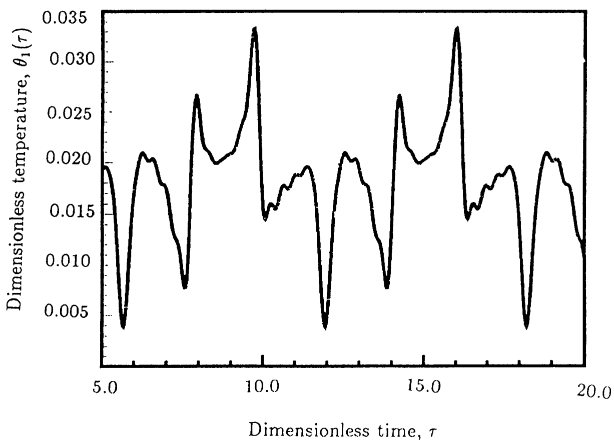

Figure 3. Time variation in the dimensionless temperature of a fixed location at the rotating rod.

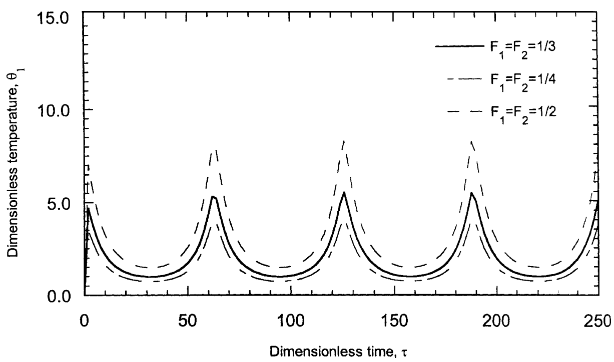

Figure 4. The effect of the fraction of energy absorbed by the rods on the time variation in the dimensionless temperature of a fixed location. $S_1 = S_2 = 10^3$, $\phi_0 = 0.1$.

Figure 3 presents the solution of equation (24) for the general case for $E(\tau)$ given in *figure 2*. It is clear from this figure that the frictional heating has a little effect on the temperature at surface of the moving rod. This is due to the slow speed of the rod and small values of S and F which are 10 and 0.1, respectively.

Figures 4 and *5* demonstrate the solution of the special case given by equation (26). *Figure 4* shows the effect of the absorbed energy by the rods on the temperature at surface of the moving rod. It is clear from this figure that as F increases, the temperature at the surface increases. The maximum increase occurs at the time of contact between the two rods. *Figure 5* shows the effect of the amount of energy generated by friction, S, on the temperature at a point on the surface of the moving rod. This figure shows that as the energy generated by friction decreases, the temperature at the surface of the moving rod decreases.

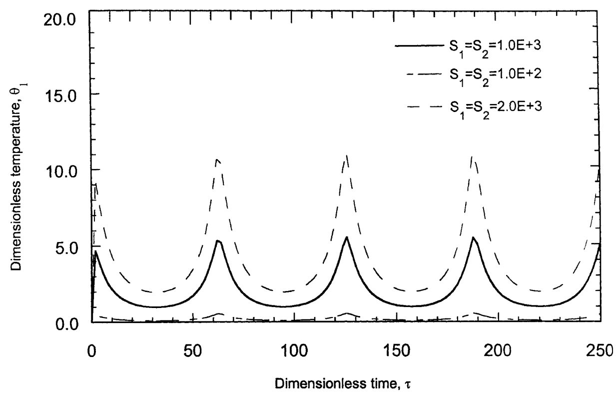

Figure 5. The effect of the energy of the frictional heating on the time variation in the dimensionless temperature of a fixed location. $F_1 = F_2 = 1/3$, $\phi_0 = 0.1$.

5. CONCLUSIONS

Using Green's function method, analytical closed form solutions, are obtained for the temperature distribution within two rods which are in contact while exhibiting sliding motion. One of the two rods is stationary and the other exhibits both oscillatory axial sliding and rotational motion. In terms of the obtained closed form solutions for the temperature distribution in both rods, the effect of different parameters on the contact area thermal behavior may be investigated. In the analysis, it is assumed that both rods have finite radii, and as a result, the semi-infinite domain assumption, in the radial direction, is not valid. A special case study is considered in which it is assumed that there is no axial sliding motion and as a result, the generated heat is only due to the rotational motion.

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APPENDIX

Equations (13)–(23) will be solved in terms of Green's function method. The appropriate Green's functions can be obtained as the solutions of the homogeneous version of equations (13)–(23), which are given as

$$\frac{\partial^2\Psi_1}{\partial Z^2}+\frac{1}{R^2}\frac{\partial^2\Psi_1}{\partial\phi^2}+\frac{\partial^2\Psi_1}{\partial R^2}+\frac{1}{R}\frac{\partial\Psi_1}{\partial R}=\frac{\partial\Psi_1}{\partial\tau} \quad (A.1)$$

$$\frac{\partial^2\Psi_2}{\partial Z^2}+\frac{1}{R^2}\frac{\partial^2\Psi_2}{\partial\phi^2}+\frac{\partial^2\Psi_2}{\partial R^2}+\frac{1}{R}\frac{\partial\Psi_2}{\partial R}=\frac{1}{\alpha_R}\frac{\partial\Psi_2}{\partial\tau} \quad (A.2)$$

with

$$\begin{aligned}&\Psi_1(0,R,Z,\phi)=\Psi_2(0,R,Z,\phi)=\theta_0\\&\frac{\partial\Psi_1}{\partial R}(\tau,0,Z,\phi)=\frac{\partial\Psi_2}{\partial R}(\tau,0,Z,\phi)=0\end{aligned} \quad (A.3)$$

$$\frac{\partial\Psi_1}{\partial R}(\tau,1,Z,\phi)+Bi_{1s}\Psi_1(\tau,1,Z,\phi)=0 \quad (A.4)$$

$$\frac{\partial\Psi_2}{\partial R}(\tau,R_1,Z,\phi)+Bi_{2s}\Psi_2(\tau,R_1,Z,\phi)=0 \quad (A.5)$$

$$\frac{\partial\Psi_1}{\partial Z}(\tau,R,0,\phi)-Bi_{1l}\Psi_1(\tau,R,0,\phi)=0 \quad (A.6)$$

$$\frac{\partial\Psi_2}{\partial Z}(\tau,R,0,\phi)-Bi_{2l}\Psi_2(\tau,R,0,\phi)=0 \quad (A.7)$$

$$\frac{\partial\Psi_1}{\partial Z}(\tau,R,H_0,\phi)+Bi_{1u}\Psi_1(\tau,R,H_0,\phi)=0 \quad (A.8)$$

$$\frac{\partial\Psi_2}{\partial Z}(\tau,R,H_3,\phi)+Bi_{2u}\Psi_2(\tau,R,H_3,\phi)=0 \quad (A.9)$$

The solutions for Ψ_1 and Ψ_2 are given as [15]

$$\begin{aligned}\Psi_1(\tau,R,Z,\phi)&\\=\frac{1}{\pi}&\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=1}^{\infty}\frac{e^{-(\beta_{1m}^2+\eta_{1p}^2)\tau}}{N(\beta_{1m})N(\eta_{1p})}\overline{R}_{1\nu}(\eta_{1m},R)\\&\cdot\overline{Z}_1(\eta_{1p},Z)\int_{\phi'=0}^{2\pi}\int_{Z'=0}^{H_0}\int_{R'=0}^{1}R'\overline{R}_{1\nu}(\beta_{1m},R')\\&\cdot\overline{Z}_1(\eta_{1p},Z')\cos\nu(\phi-\phi')\\&\cdot\theta_1(0,R',Z',\phi')\,dR'\,dZ'\,d\phi'\end{aligned} \quad (A.10)$$

$$\begin{aligned}\Psi_2(\tau,R,Z,\phi)&\\=\frac{1}{\pi}&\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{\nu=1}^{\infty}\frac{e^{-\alpha_R(\beta_{2m}^2+\eta_{2p}^2)\tau}}{N(\beta_{1m})N(\eta_{2p})}\overline{R}_{2\nu}(\eta_{2m},R)\\&\cdot\overline{Z}_2(\eta_{2p},Z)\int_{\phi'=0}^{2\pi}\int_{Z'=0}^{H_4}\int_{R'=0}^{R_1}R'\overline{R}_{2\nu}(\beta_{2m},R')\\&\cdot\overline{Z}_2(\eta_{2p},Z')\cos\nu(\phi-\phi')\\&\cdot\theta_2(0,R',Z',\phi')\,dR'\,dZ'\,d\phi'\end{aligned} \quad (A.11)$$

where $\nu=0,1,2,\ldots$ and replace π by 2π for $\nu=0$. Also,

$$\overline{R}_{1\nu}(\beta_{1m},R)=J_\nu(\beta_{1m},R)$$

$$\overline{R}_{2\nu}(\beta_{2m},R)=J_\nu(\beta_{2m},R)$$

$$\frac{1}{N(\beta_{1m})}=\frac{2\beta_{1m}^2}{J_\nu^2(\beta_{1m})([Bi_{1s}^2+\beta_{1m}^2]-\nu^2)}$$

$$\frac{1}{N(\beta_{2m})}=\frac{2\beta_{2m}^2}{R_1^2J_\nu^2(\beta_{2m}R_1)([Bi_{2s}^2+\beta_{2m}^2]-\nu^2)}$$

$$\frac{1}{N(\eta_{1p})}=\frac{2}{\left[(Bi_{1l}^2+\eta_{1p}^2)\left(H_0+\frac{Bi_{1u}}{\eta_{1p}^2+Bi_{1u}^2}\right)+Bi_{1l}\right]}$$

$$\frac{1}{N(\eta_{2p})}=\frac{2}{\left[(Bi_{2l}^2+\eta_{2p}^2)\left(H_4+\frac{Bi_{2u}}{\eta_{2p}^2+Bi_{2u}^2}\right)+Bi_{2l}\right]}$$

$$\overline{Z}(\eta_{1p},Z)=\eta_{1p}\cos\eta_{1p}Z+Bi_{1l}\sin\eta_{1p}Z$$

$$\overline{Z}(\eta_{2p},Z)=\eta_{2p}\cos\eta_{2p}Z+Bi_{2l}\sin\eta_{2p}Z$$

Finally, β_{1m}, β_{2m}, η_{1p} and η_{2p} are given as the roots of the following equations:

$$\beta_{1m} J'_\nu(\beta_{1m}) + Bi_{1s} J_\nu(\beta_{1m}) = 0$$
$$\beta_{2m} J'_\nu(\beta_{2m} R_1) + Bi_{2s} J_\nu(\beta_{2m} R_1) = 0$$
$$\tan \eta_{1p} H_0 = \frac{\eta_{1p}(Bi_{1l} + Bi_{1u})}{\eta_{1p}^2 - Bi_{1l} Bi_{1u}}$$

Now, in terms of the appropriate Green's functions, the solutions of the homogeneous problems (A.1)–(A.9) are given as

$$\begin{aligned}\Psi_1(\tau, R, Z, \phi) &= \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_0} \int_{R'=0}^{1} R' G_1\big(R, Z, \phi, \tau \big| \\ &\quad R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} \theta_1\big(0, R', Z', \phi'\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi'\end{aligned} \tag{A.12}$$

$$\begin{aligned}\Psi_2(\tau, R, Z, \phi) &= \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_4} \int_{R'=0}^{R_1} R' G_2\big(R, Z, \phi, \tau \big| \\ &\quad R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} \theta_2\big(0, R', Z', \phi'\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi'\end{aligned} \tag{A.13}$$

A comparison between equations (A.10), (A.11) and (A.12), (A.13) yields

$$\begin{aligned}G_1\big(R, Z, \phi, \tau \big| R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} &= \frac{1}{\pi} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \sum_{\nu=0}^{\infty} \frac{\mathrm{e}^{-(\beta_{1m}^2 + \eta_{1p}^2)\tau}}{N(\beta_{1m}) N(\eta_{1p})} \\ &\quad \cdot \overline{R}_{1\nu}(\beta_{1m}, R) \overline{R}_{1\nu}\big(\beta_{1m}, R'\big) \overline{Z}_{1\nu}(\eta_{1p}, Z) \\ &\quad \cdot \overline{Z}_1\big(\eta_{1p}, Z'\big) \cos \nu\big(\phi - \phi'\big)\end{aligned} \tag{A.14}$$

$$\begin{aligned}G_2\big(R, Z, \phi, \tau \big| R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} &= \frac{1}{\pi} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \sum_{\nu=0}^{\infty} \frac{\mathrm{e}^{-\alpha_R(\beta_{2m}^2 + \eta_{2p}^2)\tau}}{N(\beta_{2m}) N(\eta_{2p})} \\ &\quad \cdot \overline{R}_{2\nu}(\beta_{2m}, R) \overline{R}_{2\nu}\big(\beta_{2m}, R'\big) \overline{Z}_2(\eta_{2p}, Z) \\ &\quad \cdot \overline{Z}_2\big(\eta_{2p}, Z'\big) \cos \nu\big(\phi - \phi'\big)\end{aligned} \tag{A.15}$$

The desired Green's functions are obtained by replacing τ by $\tau - \tau^*$ in equations (A.14) and (A.15) to yield

$$\begin{aligned}G_1\big(R, Z, \phi, \tau \big| R', Z', \phi', \tau^*\big) &= \frac{1}{\pi} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \sum_{\nu=0}^{\infty} \frac{\mathrm{e}^{-(\beta_{1m}^2 + \eta_{1p}^2)(\tau-\tau^*)}}{N(\beta_{1m}) N(\eta_{1p})} \\ &\quad \cdot \overline{R}_{1\nu}(\beta_{1m}, R) \overline{R}_{1\nu}\big(\beta_{1m}, R'\big) \overline{Z}_1(\eta_{1p}, Z) \\ &\quad \cdot \overline{Z}_1\big(\eta_{1p}, Z'\big) \cos \nu\big(\phi - \phi'\big)\end{aligned} \tag{A.16}$$

$$\begin{aligned}G_2\big(R, Z, \phi, \tau \big| R', Z', \phi', \tau^*\big) &= \frac{1}{\pi} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \sum_{\nu=0}^{\infty} \frac{\mathrm{e}^{-\alpha_R(\beta_{2m}^2 + \eta_{2p}^2)(\tau-\tau^*)}}{N(\beta_{2m}) N(\eta_{2p})} \\ &\quad \cdot \overline{R}_{2\nu}(\beta_{2m}, R) \overline{R}_{2\nu}\big(\beta_{2m}, R'\big) \overline{Z}_2(\eta_{2p}, Z) \\ &\quad \cdot \overline{Z}_2\big(\eta_{2p}, Z'\big) \cos \nu\big(\phi - \phi'\big)\end{aligned} \tag{A.17}$$

In terms of these Green's functions, solutions for the non-homogeneous problems (13)–(23) are given as

$$\begin{aligned}\theta_1(\tau, R, Z, \phi) &= \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_0} \int_{R'=0}^{1} R' G_1\big(R, Z, \phi, \tau \big| \\ &\quad R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} \theta_1\big(0, R', Z', \phi'\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi' \\ &\quad + \int_{\tau^*=0}^{\tau} \mathrm{d}\tau^* \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_0} \int_{R'=0}^{1} R' G_1\big(R, Z, \phi, \tau \big| \\ &\quad R', Z', \phi', \tau^*\big) S_1 F_1 \delta\big(R' - 1\big) \delta\big(\phi' - \phi_0 \tau^*\big) \\ &\quad \cdot \delta\big(Z' - E\big(\tau^*\big)\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi'\end{aligned} \tag{A.18}$$

$$\begin{aligned}\theta_2(\tau, R, Z, \phi) &= \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_4} \int_{R'=0}^{R_1} R' G_2\big(R, Z, \phi, \tau \big| \\ &\quad R', Z', \phi', \tau^*\big)\big|_{\tau^*=0} \theta_2\big(0, R', Z', \phi'\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi' \\ &\quad + \alpha_R \int_{\tau^*=0}^{\tau} \mathrm{d}\tau^* \int_{\phi'=0}^{2\pi} \int_{Z'=0}^{H_4} \int_{R'=0}^{R_1} R' \\ &\quad \cdot G_2\big(R, Z, \phi, \tau \big| R', Z', \phi', \tau^*\big) S_2 F_2 \delta\big(R' - R_1\big) \\ &\quad \cdot \delta(\phi') \delta\big(Z' - H_3\big)\, \mathrm{d}R'\, \mathrm{d}Z'\, \mathrm{d}\phi'\end{aligned} \tag{A.19}$$

Substitute for G_1 and G_2 from equations (A.16) and (A.17) into equations (A.18) and (A.19), with the assumption of uniform initial conditions within the two rods, i.e., $\theta_1(0, R, Z, \phi) = \theta_2(0, R, Z, \phi) = \theta_0$, yields the final forms of the solution as given by equations (24) and (25).